

Teaching Multiplication and Division Facts to the Whole- to -Part, Visual Learner:

A Home Study Guide to Developing Fluency With Math Facts.

Christopher L. Woodin, Ed. M. 2006

Efficient Fact Learning At Landmark School E.M.S.

• Fluency Involves Accuracy and Efficiency.

Fluency is achieved through the accurate repetition of knowledge, in this case math fact “sentences.” It is very important to drill for fluency only after the student is able to produce a math fact accurately. The student should never be placed in a situation that forces them to guess and produce a fact error. Verbalizing inaccurate fact sentences compromises the learning process and leads to frustration. Students should always have the ability to either refer to the correct answer and read it, use a learned strategy to create the answer, or say, “pass” and have the correct answer supplied before expressing the entire fact sentence. A great deal of time is spent in math class developing strategies that promote awareness of fact family patterns, and empower students to construct accurate fact sentences. Progress in efficiency cannot be achieved without accuracy.

• Provide Wait Time For Verbal Expression.

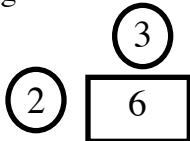
Many students with language-based learning disabilities need ample time to formulate and express their responses. Learned strategies that help your child may involve visualizing a graphic organizer, or referring to a pattern inherent to the fact family. This active reasoning demands processing time. Allow the student to make a response or say “pass” before attempting to cue or prompt a response. Responses may take 15 seconds to formulate. Try to be patient and not let your impatience or frustration rub off on your child. It doesn’t help- it just adds to their anxiety level. Time a 15 second period of silence prior to working with your child. It seems like an eternity, but time spent processing in a thoughtful manner will eventually produce an accurate, and increasingly efficient response. In this manner, the efficient expression of math facts may be integrated with the visual, kinesthetic, or semantic (meaning-based) patterns that have been associated with that fact family during math class.

• Multiplication Facts Should Be Practiced in Conjunction With Their Division Counterparts.

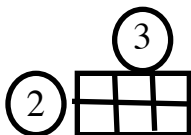
Learning facts in a relational context that prompts the student to recognize and express them in both multiplication and division formats is very important. Facts may be expressed with four fact sentences: two multiplication, and two division (with the exception of perfect squares). The entire fact family is modeled with a rectangular matrix diagram that dovetails visually with the area model of multiplication, as well as traditional division notation (see figure1 below). Gaining exposure to the dynamic expression of fact knowledge allows students to develop necessary flexibility with their knowledge base. Creating these four related math sentences from a completed diagram provides a relatively error free opportunity to practice. It also involves processing the three fact elements rather than merely repeating them. Time used to hold the fact information in short term memory while formulating the four related facts may facilitate transfer of the fact to long term memory. Another strategy to promote the transfer of facts from short term to long term memory involves the constant time delay procedure that is described below.

figure1

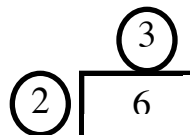
Rectangular Matrix Diagram



Area Model



Division Notation



Four Related Facts

$$\begin{aligned} 2 \times 3 &= 6 \\ 3 \times 2 &= 6 \\ 6 \div 2 &= 3 \\ 6 \div 3 &= 2 \end{aligned}$$

• Constant Time Delay

This process involves showing the student a flash card, then having the student raise his hand or point to an answer when he is able. After waiting four seconds, clap your hands to prompt the student to respond with the missing factor or product. If his answer is correct, have him then recite a complete multiplication sentence and a related division sentence. The imposed delay encourages the student to internally recite the fact. Research has shown this procedure to facilitate long term retention of facts in some children.

Order Of Fact Instruction

• Cadenced facts: x2, x10, x5, x1

These facts are the most important, and most frequently required facts. They are called cadenced facts as the products may be skip counted with a metered beat- in the manner of a drum cadence. These four fact families comprise 64% of the 100 multiplication facts. They are necessary for multidigit computation, telling time, developing base ten relationships, and fraction simplification. They also lend themselves to many semantic associations that may be modeled and expressed through word problems. Skip counting helps familiarize the student with the products; however, it is not a reliable or efficient means to produce facts. The process of defining factors of products by skip counting may place overwhelming demands on the student's auditory processing abilities. Students often miscount the number of beats of the cadence, or arrive at the correct product without ever rehearsing the entire fact sentence. As a result, they become rigidly dependent on this counting strategy and fail to make progress toward recognizing and retrieving the entire multiplication fact sentence, or a division correlate.

Time spent producing the cadenced facts in an effortless, fluent manner has a marked impact on math learning. These are the fact families that need regular drill and practice to develop fluency. Time spent at home working on these fact families will have the greatest positive impact on your child.

Divisibility Rules

The x2, x10, x5, and x1 fact families have products that are easily recognizable. As a result, divisibility rules have been developed to define them. These rules are helpful in that they provide an efficient means to recognize and accept, or exclude and reject a product from each family. Prior to practicing a multiplication fact family, it is helpful to review its accompanying divisibility rule. The rule will help to constrain the student's products to a pool of acceptable answers. See figure 2 for a visual depiction of the following rules. Highlighting each fact family on a 1-100 chart provides an opportunity for the student to see these patterns.

- Multiples (products) of two are always even numbers. They have a 0,2,4,6, or 8 in their one's place.
- Multiples of ten always have a zero in their one's place.
- Even multiples of five always end in zero while odd multiples end in five.
- Multiples of one match their multiplying factor.

Figure 2

<u>Multiples of 2</u>					<u>Multiples of 5</u>		<u>Multiples of 10</u>			<u>Multiples of 1</u>									
2	4	6	8	10	5	10	10	1	2	3	4	5	6	7	8	9	10		
12	14	16	18	20	15	20	20	11	12	13	14	15	16	17	18	19	20...		
					25	30	30												
					35	40	40												
					45	50	50...												

Practicing at Home

After your child has made sufficient progress understanding each successive fact family, flash card sheets and related activities will be sent home. Additional copies of these materials will be available on the following web site: <http://cwoodinmathfacts.tripod.com>. They are downloadable to any computer as a PDF file. Practice only those fact families and activities that have already been addressed in school in order to avoid any confusion.

Use Hands to Model Two-Times Facts and Place Value

Have students hold out their left hand with fingers extended.

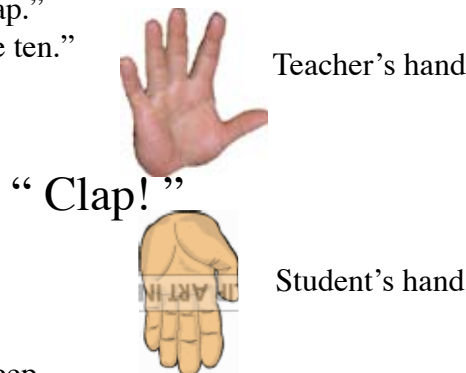
* Make sure that their hand is within their visual field- so that they can see it.

Teacher: "You are showing me five fingers, one time."

The teacher should model this by mirroring their image - extending his right hand as he faces the student.

"High five" the student to make a "clap."

Teacher: "This is five, two times or one ten."



The teacher should ask the student to keep the student's left fingers extended.

Teacher: "Now show me seven fingers by sticking out two more fingers (on your right hand)."

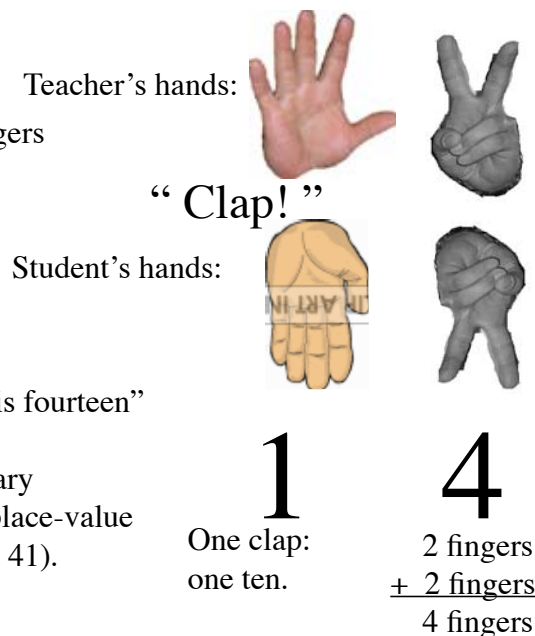
Teacher: "You are showing me seven one time."

As the teacher faces the student, the teacher models this by mirroring their image:

The teacher extends the five fingers on his right hand, as well as two from his left hand.

Teacher: "This is fourteen. It is seven two times. Two times seven is fourteen"

This hand model represents the product 14 within the students primary reference frame. This provides the student with a internally-based place-value template to help him encode the digits in their proper order (14, not 41).



Shootout Game

Start the game in the ready position: The teacher is extending all five of the fingers on his right hand. This hand should hang at belt level- like an old west gun fighter about to "draw" his sidearm.

The student should assume a similar posture -have his five left hand fingers extended with his left hand hanging at his side.

The teacher should call out a number from 5 to 9. That is the signal to "draw."

Both teacher and student rush to display the quantity of fingers to match the number called-out- pointing their hands at one another.

The pairs of hands will create a two times fact that should be verbalized by the winner of this finger drawing contest. The other person should state a related fact.


For example: The teacher calls out, "seven." Both teacher and student "shoot-out" seven fingers as pictured above. The teacher and student should move toward each other, clap their five-fingered hands, and identify the number of additional fingers that will comprise the one's digit of the product (in this case $2+2=4$)


If the student gets his seven extended first, he must verbalize that $2x$ fact: " $14 = 2 \times 7$," or " $7 \times 2 = 14$."


The teacher should respond with a related division fact, " $14 \div 2 = 7$."


2x Facts: Cut out these flash cards on the dotted lines.

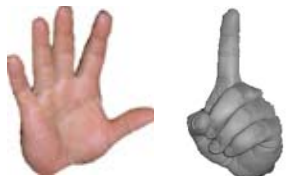
Students mirror the fingers to produce "two times" the quantity of fingers, then verbalize the four related facts.


2 

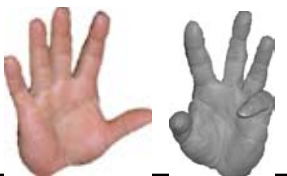
2 


2 

2 

2 

2 

2 

2 

More Activities to learn 2x Facts

Fact Twister

This activity reinforces number facts as well as directionality. It can be played individually or in small groups. Write the products of the fact family you are addressing on paper plates. Arrange the paper plates on the floor in front of the student. Have the students answer your fact questions by touching the correct plate with the appendage you name.

•• Example

Teacher, “Put your right foot on the product of two times five.”

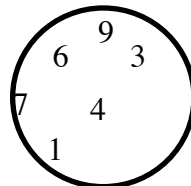
When a student correctly performs the task, signal success by asking for the parroted fact, $2 \times 5 = 10$; the commuted multiplication fact, $5 \times 2 = 10$; or one of the two related division facts, $10 \div 5 = 2$ or $10 \div 2 = 5$.

Rad Minute

After the student writes his name, date and day on the Rad Minute sheet, have him fold the paper along the dotted line. The first order of business is to have the student fill-in the products from top to bottom. This may be achieved by skip counting by twos: “2, 4, 6...” etc. Next, have the student check all of the products using the 2x divisibility rule:

“All of the products must have a 0,2,4,6 or 8 in their one’s place.”

The teacher should also check the products for accuracy within each fact. For instance, be on the lookout for errors like $2 \times 7 = 18$. A check for divisibility will not pick up this type of error. After that, have the student copy each math fact in its entirety. Each fact should be written from left to right like a sentence. Do not allow the student to write in columns: all of the first digits of the facts, then all of the “x” signs, etc. The object is to have the student rehearse each fact as a unit so that it may be stored, then later retrieved from auditory memory. See the Rad Minute sheet on the following page.



Fact Ball

After practicing a multiplication fact family using flash cards or another production task, play a game of fact ball. Write numbers on a soft, light ball - perhaps a tennis ball. Toss the ball to the student. The student should use the first number that he sees on the ball to begin the fact sentence. Initially, write only those factors that are most familiar, perhaps 1,2,5,10. Add more factors as the student gains confidence with harder facts.

For example, the teacher tosses the ball to Johnny who catches it with one hand. Have Johnny read the number from the ball that is closest to his thumb. “Six,” says Johnny. “Ok,” says the teacher, “say a 2 times sentence starting with 6...”

Johnny says, “Six times two is 12.” The teacher responds with a related fact, “Two times six is 12.”

The teacher may also choose to model one of the two related division facts: $12 \div 2 = 6$, or $12 \div 6 = 2$. Johnny then tosses or rolls the ball to the teacher and the roles are reversed.

Make sure that the student has the ability to look up each correct math fact if necessary. The student should never be forced to guess at an answer. If unsure of a fact, prompt the student to either wait and think, look it up, or say “pass”- and have the answer supplied by the teacher. The student should then rehearse the fact by saying it in its entirety.

RAD MINUTE

- 1) Fill in blanks.
- 2) Check with divisibility rules.
- 3) Copy each number sentence.

Correct

20

Warm -up

Copy: $1 \times 2 =$

 $2 \times 2 =$

$3 \times 2 =$

$4 \times 2 =$

$5 \times 2 =$

$6 \times 2 =$

$7 \times 2 =$

$8 \times 2 =$

$9 \times 2 =$

$10 \times 2 =$

One Minute Quiz

$3 \times 2 =$

$8 \times 2 =$

$2 \times 2 =$

$6 \times 2 =$

$9 \times 2 =$

$4 \times 2 =$

$1 \times 2 =$

$7 \times 2 =$

$10 \times 2 =$

$5 \times 2 =$